Pre-resonant Charmonium - Nucleon Cross Section in the Model of the Stochastic Vacuum

H.G. Dosch[a], F.S. Navarra [b] and M. Nielsen [b]

[a]Institut für Theoretische Physik, Universität Heidelberg
Philosophenweg 16, D-6900 Heidelberg, Germany

[b]Instituto de Física, Universidade de São Paulo,
C.P. 66318, 05315-970 São Paulo, SP, Brazil

We calculate the nonperturbative charmonium - nucleon cross sections with the model of the stochastic vacuum which has been succesfully applied in many high energy reactions. We also give a quantitative discussion of pre-resonance formation and medium effects.

1. Introduction

Charmonium nucleon cross sections are of crucial importance in the context of Quark Gluon Plasma (QGP) physics [1,2]. One needs to know the cross section $\sigma_{c\overline{c}-N}$ in order to explain nuclear suppression of J/Ψ in terms of ordinary absorption by nucleons without assuming a so called "deconfining regime". Estimates using perturbative QCD give values which are too small to explain the observed absorption conventionally, but they are certainly not reliable for that genuine nonperturbative problem. A nonperturbative estimate may be tried by applying vector dominance to J/Ψ and Ψ' photoproduction. In this way a cross section of $\sigma_{J/\psi} \simeq 1.3$ mb for $\sqrt{s} \simeq 10$ GeV and $\sigma_{\psi'}/\sigma_{J/\psi} \simeq 0.8$ has been obtained [3,4]. A more refined multichannel analysis [3] leads to $\sigma_{J/\psi} \simeq 3-4$ mb.

The fact that the absorption cross section seems to be nearly the same both for J/ψ and ψ' has been interpreted as meaning that what is really absorbed is rather a pre-resonant $c-\overline{c}$ state and not the physical particles. The size of this state has been estimated to be [5]

$$r_8 = \frac{1}{\sqrt{2m_c\Lambda_{QCD}}} = 0.2 - 0.25\text{fm} \qquad (1)$$

and its cross section was then calculated with short distance QCD. A value of $\sigma_8 \simeq 5.6-6.7$ mb was found.

In this note we calculate the pre-resonant $c-\overline{c}$-nucleon cross sections in the model of the stochastic vacuum (MSV) [6–9]. It has been applied to a large number of hadronic and photoproduction processes with remarkably good success.

2. The Model of the Stochastic Vacuum

The basis of the MSV is the calculation of the scattering amplitude of two colourless dipoles [10,9] based on a semiclassical treatment developped by Nachtmann [11]. The dipole-dipole scattering amplitude is expressed as the expectation value of two Wegner-Wilson loops with lightlike sides and transversal extensions $\vec{r}_{t1}$ and $\vec{r}_{t2}$ respectively. This leads to a profile function $J(\vec{b},\vec{r}_{t1},\vec{r}_{t2})$ from which hadron-hadron scattering amplitudes are obtained by integrating over different dipole sizes with the transversal densities of the hadrons as weight functions according to

$$\sigma^{tot}_{(c-\overline{c})-N} = \int d^2b\, d^2r_{t1}\, d^2r_{t2} \times \rho_{(c-\overline{c})-N}(\vec{r}_{t1})\,\rho_N(\vec{r}_{t2})\,J(\vec{b},\vec{r}_{t1},\vec{r}_{t2}) \qquad (2)$$

Here $\rho_{(c-\overline{c})-N}(\vec{r}_{t1})$ and $\rho_N(\vec{r}_{t2})$ are the transverse densities of the pre-resonant charmonium state and nucleon respectively.

The basic ingredient of the model is the gauge invariant correlator of two gluon field strength tensors. The latter is characterized by two con-

stants: the value at zero distance, the gluon condensate $< g^2FF >$, and the correlation length a. We take these values from previous applications of the model [10] (and literature quoted there): $< g^2FF >= 2.49$ GeV^4 and $a = 0.346$ fm. The wave functions of the proton have been determined from proton-proton and proton-antiproton scattering respectively. It turns out that the best description for the nucleon transverse density is given by that of a quark - diquark system with transversal distance $\vec{r}_t$ and density:

$$\rho_N(\vec{r_t}) = |\Psi_p(\vec{r}_t)|^2 = \frac{1}{2\pi}\frac{1}{S_p^2}\, e^{-\frac{|\vec{r}_t|^2}{2S_p^2}} \quad . \tag{3}$$

The value of the extension parameter, $S_p = 0.739$ fm, obtained from proton-proton scattering agrees very well with that obtained from the electromagnetic form factor in a similar treatment.

We start estimating the cross section in the case where the $c - \overline{c}$ pair is already in the physical J/ψ or ψ' states. The physical wave functions can be obtained in two different approaches: 1) A numerical solution of the Schroedinger equation with the standard Cornell potential [12]:

$$V = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \tag{4}$$

2) A Gaussian wave function determined by the electromagnetic decay width of the J/Ψ which has been used in a previous investigation of J/Ψ photoproduction [10].

The linear potential can be calculated in the model of the stochastic vacuum which yields the string tension:

$$\sigma = \frac{8\kappa}{81\pi} < g^2FF > a^2 \; = 0.179\,\mathrm{GeV}^2 \tag{5}$$

where the parameter κ has been detemined in lattice calculations to be $\kappa = 0.8$ [13].

The other parameters, the charmed (constituent) mass and the (frozen) strong coupling can be adjusted in order to give the correct J/Ψ and Ψ' mass difference and the J/ψ decay width:

$$m_c = 1.7\mathrm{GeV} \qquad \alpha_\mathrm{s} = 0.39 \tag{6}$$

We also use the standard Cornell model parameters [12]:

$$\alpha_s = 0.39 \; \sigma = 0.183 \text{ GeV}^2 \; m_c = 1.84 \text{ GeV} \tag{7}$$

From the numerical solution $\psi(|\vec{r}|)$ of the Schroedinger equation the transversal density is projected:

$$\rho_{J/\Psi}(\vec{r}_t) = \int \left|\psi(\sqrt{\vec{r}_t^{\;2} + r_3^2})\right|^2 dr_3 \tag{8}$$

where $\vec{r}_t$ is the J/Ψ transversal radius.

Given the values of α_s, σ and m_c we solve the non-relativistic Schroedinger equation numerically, obtain the wave function, compute the transverse wave function and plugg it into the MSV calculation [9].

In the pre-resonance absorption model, the pre-resonant charmonium state is either interpreted as a color-octet, $(c\overline{c})_8$, and a gluon in the hybrid $(c\overline{c})_8 - g$ state, or as a coherent $J/\Psi - \Psi'$ mixture. For the pre-resonant state we use a gaussian transverse wave function, as in Eq. (3), to represent a state with transversal radius $\sqrt{< r_t^2 >} = \sqrt{2}S_\psi$. S_ψ is the pre-resonance extension parameter analogous to S_p. The relation between the average transverse radius and the average radius is given by:

$$\sqrt{< r_t^2 >} \simeq 0.82\,\sqrt{< r^2 >} \tag{9}$$

With the knowledge of the wave functions and transformation properties of the constituents we can compute the total cross section given by the MSV. The resulting nucleon - pre-resonant charmonium state cross section will be different if the pre-resonant charmonium state consists of entities in the adjoint representation (as $(c\overline{c})_8 - g$) or in the fundamental representation (as a $J/\Psi - \Psi'$ mixture), the relation being:

$$\sigma_{\mathrm{adjoint}} = \frac{2N_C^2}{N_C^2 - 1}\sigma_{\mathrm{fundamental}} \tag{10}$$

with $N_C = 3$

3. Results

The results are shown in Table I. In this table $\sqrt{< r^2 >}$ is the root of the mean square distance of quark and antiquark. Wave function A)

is the one obtained with the parameters given by Eqs. (5) and (6). Wave function B) corresponds to the standard Cornell model parameters, Eq. (7). Wave function C) gives the result for the $J/\Psi - N$ cross section obtained with the weighted average of the longitudinally and transversely polarized J/Ψ wave functions from [10] with transversal sizes $\sqrt{< r_t^2 >} = 0.327$ fm and 0.466 fm.

Wave function	$\sqrt{< r^2 >}$ fm	σ_{tot} [mb]
$J/\Psi(1S)$		
A	0.393	4.48
B	0.375	4.06
C		4.69
$\Psi(2S)$		
A:	0.788	17.9

TABLE I $J/\Psi - N$ and $\Psi' - N$ cross section

Averaging over our results for different wave functions, our final result for the $J/\Psi - N$ cross section is

$$\sigma_{J/\psi - N} = 4.4 \pm 0.6 \,\mathrm{mb} \qquad (11)$$

The error is an estimate of uncertainties coming from the wave function and the model. Other nonperturbative calculations of the $J/\Psi - N$ cross section were presented in ref.[14], where the value $\sigma_{J/\psi - N} = 3.6$ mb was found, and in ref.[15], which reported $\sigma_{J/\psi - N} = 2.8$ mb. Our result is somewhat larger but still in agreement with these numbers. For Ψ' our cross section is slightly smaller than $\sigma_{\psi' - N} = 20.0$ mb, obtained in [14] but larger than $\sigma_{\psi' - N} = 10.5$ mb as found in [15].

In Table II we show the results for the absorption cross section of the pre-resonant charmonium state, interpreted as the color-octet, $(c\overline{c})_8 - g$ and as the coherent $J/\Psi - \Psi'$ mixture for different values of the average squared radius.

$\sqrt{< r^2 >}$ (fm)	$\sigma_{c\bar{c}}$ (mb)	$\sigma_{(c\bar{c})_8 - g}$ (mb)
0.24	1.79	4.02
0.31	2.76	6.21
0.37	3.96	8.91
0.43	5.30	11.92
0.49	6.81	15.32
0.55	8.50	19.12
0.61	10.28	23.13

TABLE II The cross section pre-resonant charmonium-nucleon

From our results we can see that a cross-section $\sigma_{\psi}^{abs} \simeq 6-7$ mb, needed to explain the J/Ψ and Ψ' suppression in p-A collisions in the pre-resonance absorption model [16,17], is consistent with a pre-resonant charmonium state of size $\simeq 0.50 - 0.55$ fm if it is a $J/\Psi - \Psi'$ mixture or $\simeq 0.30 - 0.35$ fm for a $(c\bar{c})_8 - g$ state. This last estimate can be compared with r_8 and σ_8 quoted above. For $\sqrt{< r^2 >} = r_8 \simeq 0.25$ fm we obtain a cross section of 4 mb instead of 6.7 mb, as obtained in [5]. Inspite of the uncertainty in these numbers we can see that our calculation leads to smaller values for the cross sections. Alternatively, we may reverse the argument and say that the pre-resonant octet state must have a larger radius than previously estimated. This seems to be unlikely, especially in view of the estimates of sizes and lifetimes performed in [15]. This conclusion will become even stronger with the inclusion of medium effects.

In a hadronic medium, the QCD vacuum parameters may change. Indeed, lattice calculations [13] show that both the correlation length and the quark and gluon condensates tend to decrease in a dense (or hot) medium. The first consequence, largely explored in cross section calculations, is the change of hadron masses [18]. The second consequence is a reduction of the string tension, σ, which leads to two competing effects, which can be quantitatively compared in the MSV. On one hand the cross section tends to decrease strongly when the gluon condensate or the correlation length decrease. On the other hand, when the string tension is reduced the $c - \overline{c}$ state becomes less confined and will have a larger

radius, which, in turn, would lead to a larger cross section for interactions with the nucleons in the medium. In the MSV we can determine which of these effects is dominant.

Although all the calculations are done numerically, we can parametrize the dependence of the cross sections on some specific quantities. We have therefore the following three possibilities to express the cross section as a function of the string tension, σ, the correlation length, a, and the gluon condensate, $< g^2 FF >$ [19]:

$$\sigma_{\psi\, N} \propto \left\{ \begin{array}{c} \sigma^{5/6} a^{5/2} \\ \sigma^{25/12} < g^2 FF >^{-5/4} \\ < g^2 FF >^{5/6} a^{25/6} \end{array} \right. \tag{12}$$

From the equation above we see that the final effect of the medium is a reduction in the cross section. Using the values of the correlation length and the gluon condensate reduced by 10%: $a =$ 0.31 fm , $\langle g^2 FF \rangle = 2.25$ GeV^4, we obtain a 40% reduction in the cross sections.

Taking this reduction into account the absorption cross sections obtained both for the physical J/ψ (Eq. (11)) and for the $J/\psi - \psi'$ mixture (second column in Table II) are smaller than the ones needed in Refs. [15] , [16] or [17] to explain experimental data. The absorption cross section of the hybrid $(c\bar{c})_8 - g$ state, even after the inclusion of medium effects, is still compatible (although somewhat small) with the values quoted in the mentioned papers.

To summarize, we calculated the nonperturbative $J/\Psi - N$ and $\Psi' - N$ cross sections with the MSV. We obtain $\sigma_{J/\psi\, N} \sim 4$ mb and $\sigma_{\psi'\, N} \sim 18$ mb. An interesting prediction of the MSV is the strong depedence on the parameters of the QCD vacuum which will most likely lead to a drastic reduction of them at higher temperatures and perhaps also at higher densities.

REFERENCES

1. *Proceedings of the Quark Matter '97 Conference*, *Nucl. Phys.* **A638** (1998).
2. B. Müller, "Quark Matter '99 - Theoretical Summary. What Next?", nucl-th/9906029.
3. J.Hüfner and B.Z.Kopeliovich, *Phys. Lett.* **B426** (1998) 154 and references therein.
4. L. Frankfurt and M. Strikman, *Nucl. Phys.* **B250** (1985) 147.
5. D. Kharzeev and H. Satz, *Phys. Lett.* **B366** (1996) 316; **B356** (1995) 365; **B334** (1994) 155.
6. H.G. Dosch, *Phys. Lett.* **B190** (1987) 177.
7. H.G. Dosch and Y.A. Simonov, *Phys. Lett.* **B205** (1988) 339.
8. Y.A. Simonov, *Nucl. Phys.* **B307** (1988) 512.
9. H.G. Dosch, E. Ferreira, A. Krämer, *Phys. Rev.* **D50** (1994) 1992 and references therein.
10. H.G. Dosch, T. Gousset, G. Kulzinger and H.J. Pirner, *Phys. Rev.* **D55** (1997) 2602.
11. O. Nachtmann, *Lectures, Schladming (Austria)*, hep-ph-9609365 (1996); *Annals Phys.* **209** (1991) 436.
12. E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.-M. Yan, *Phys. Rev.* **D17** (1978) 3090; *Phys. Rev.* **D21** (1980) 203.
13. A. Di Giacomo, E. Meggiolaro and H. Panagopolos, *Nucl. Phys. Proc. Suppl.* **A54** (1997) 343.
14. L. Gerland *et al.*, *Phys. Rev. Lett.* **81** (1998) 762.
15. Y.B. He, J. Hüfner and B.Z. Kopeliovich, "J/Ψ and ψ' total cross sections and formation times from data for charmonium suppression in $p - A$ collisions", hep-ph/9908243.
16. D. Kharzeev, C. Lorenço, M. Nardi and H. Satz, *Z. Phys.* **C74** (1997) 318.
17. C. Spieles *et al.*, "Modelling J/Ψ production and absorption in a microscopic nonequilibrium approach", hep-ph/9902337 and references therein.
18. F.S. Navarra and C.A.A. Nunes, *Phys. Lett.* **B356** (1995) 439.
19. "Charmonium - Hadron Cross Section in a Nonperturbative QCD Approach", H.G. Dosch, F.S. Navarra, M. Nielsen and M. Rüter, hep-ph/9908274